



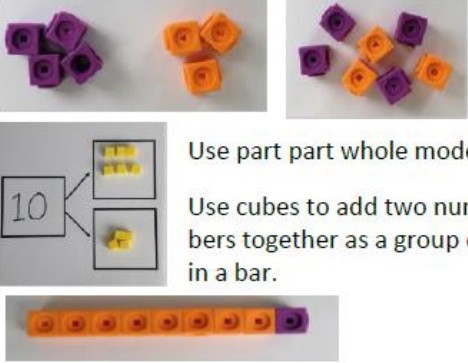
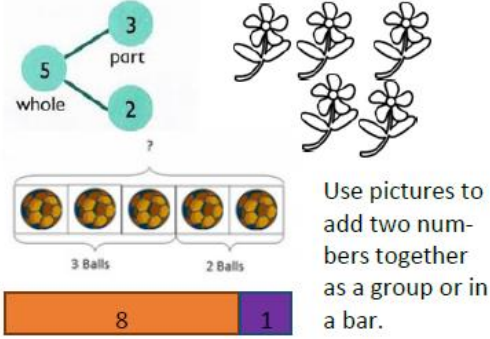


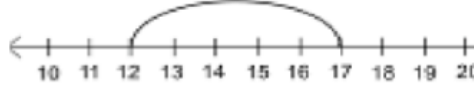
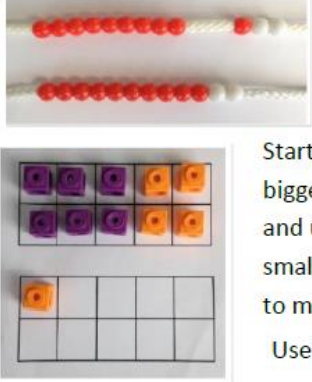
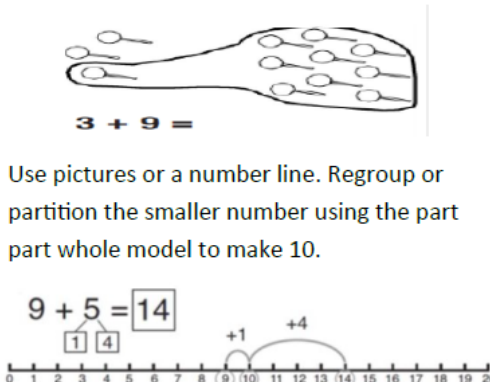

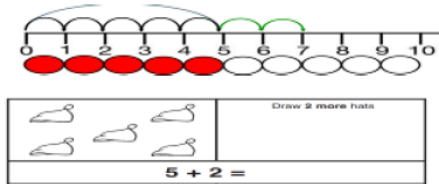
Numeracy Calculation Policy

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added.

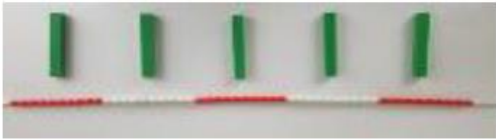
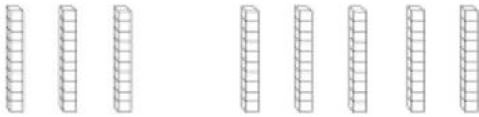
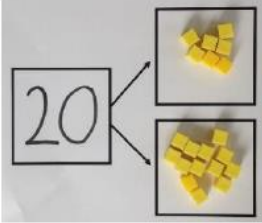
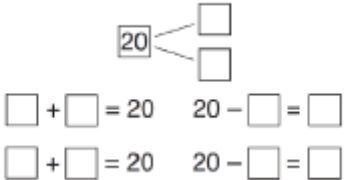
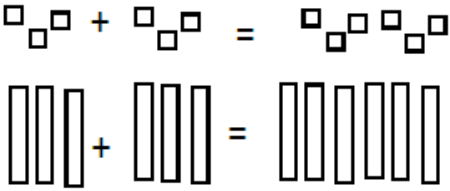
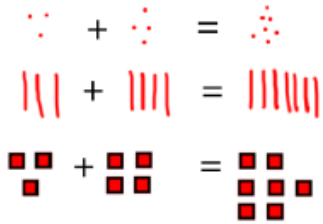


It is a working document and will be revised and amended as necessary.

Updated November 2019

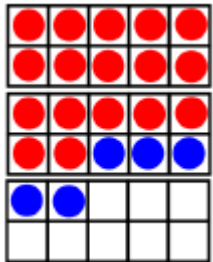
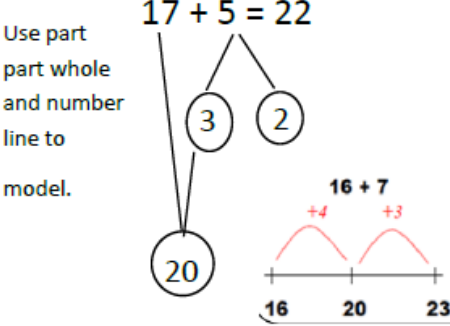
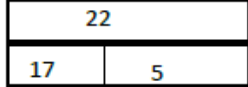

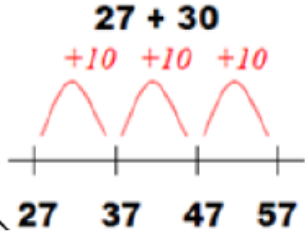

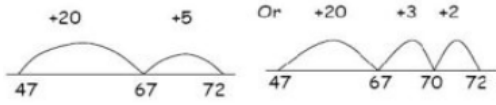
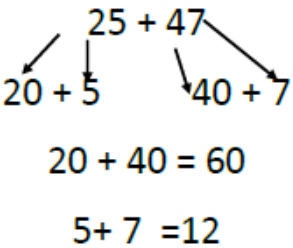

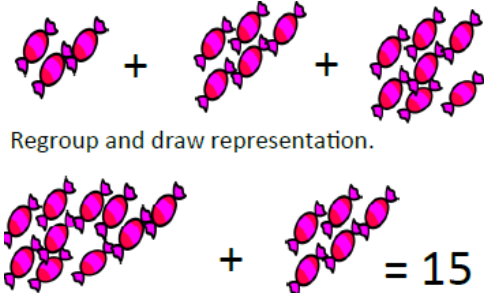
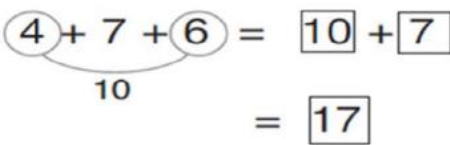
Y1 Addition +

| Objective & Strategy | Concrete | Pictorial | Abstract |
|--------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Combining two parts to make a whole: part- whole model |  <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p> |  <p>Use pictures to add two numbers together as a group or in a bar.</p> | $4 + 3 = 7$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> |
| Starting at the bigger number and counting on |  <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p> | $12 + 5 = 17$  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p> | $5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p> |
| Regrouping to make 10. <i>This is an essential skill for column addition later.</i> |  <p>$6 + 5 = 11$</p> <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p> |  <p>$3 + 9 =$</p> <p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p> <p>$9 + 5 = 14$</p> | $7 + 4 = 11$ <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p> |
| Represent & use number bonds and related subtraction facts within 20 |  <p>2 more than 5.</p> |  <p>$5 + 2 =$</p> | <p>Emphasis should be on the language</p> <p>'1 more than 5 is equal to 6.'</p> <p>'2 more than 5 is 7.'</p> <p>'8 is 3 more than 5.'</p> |

Y2 Addition +

| Objective & Strategy | Concrete | Pictorial | Abstract | | | | |
|-----------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|----|---|--|
| Adding multiples of ten | <p>50 = 30 + 20</p>  <p>Model using base 10 and bead strings</p> |  <p>3 tens + 5 tens = _____ tens 30 + 50 = _____</p> <p>Use representations for base ten.</p> | $20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$ | | | | |
| Use known number facts Part Part Whole Model |  <p>Children explore ways of making numbers within 20</p> |  <p>$\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$</p> | $\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$ | | | | |
| Using known facts |  | <p>Children draw representations of H, T and O</p>  | $3 + 4 = 7$ leads to $30 + 40 = 70$ | | | | |
| Bar model |  <p>$3 + 4 = 7$</p> |  <p>$7 + 3 = 10$</p> | <table border="1" data-bbox="1709 1098 2168 1214"> <tr> <td>23</td> <td>25</td> </tr> <tr> <td colspan="2">?</td> </tr> </table> <p>$23 + 25 = 48$</p> | 23 | 25 | ? | |
| 23 | 25 | | | | | | |
| ? | | | | | | | |

Y2 Addition +

| Objective & Strategy | Concrete | Pictorial | Abstract |
|----------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Add a two digit number and ones</p> |  <p>$17 + 5 = 22$</p> <p>Use a tens frame to make 'magic ten'</p> <p>Children explore the pattern</p> <p>$17 + 5 = 22$ $27 + 5 = 32$</p> | <p>Use part part whole and number line to model.</p> <p>$17 + 5 = 22$</p>  | <p>$17 + 5 = 22$</p> <p>Explore related facts</p> <p>$17 + 5 = 22$</p> <p>$5 + 17 = 22$</p> <p>$22 - 17 = 5$</p> <p>$22 - 5 = 17$</p>  |
| <p>Add a 2 digit number and tens</p> |  <p>$25 + 10 = 35$</p> <p>Explore that the ones digit does not change</p> | <p>$27 + 30$</p>  | <p>$27 + 10 = 37$</p> <p>$27 + 20 = 47$</p> <p>$27 + \square = 57$</p> |
| <p>Add two 2-digit numbers</p> | <p>Model using base 10, place value counters and Numicom</p>  |  <p>Use number line and bridge ten using part whole if necessary.</p> | <p>Partition to add</p>  <p>$20 + 40 = 60$</p> <p>$5 + 7 = 12$</p> |
| <p>Add three 1-digit numbers</p> |  <p>Combine to make 10 first if possible, or bridge 10 then add third digit</p> | <p>Regroup and draw representation.</p>  <p>$4 + 7 + 6 = 15$</p> | <p>Combine the two numbers that make/bridge ten then add on the third.</p>  <p>$4 + 7 + 6 = 10 + 7 = 17$</p> |

Y3 Addition +

Objective & Strategy

Column addition:
no regrouping or
exchanges

Concrete

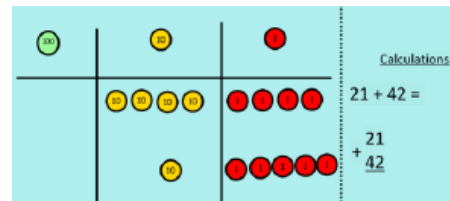


Model using
base 10 or
Numicom



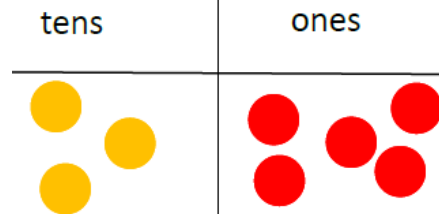
Add
together
the ones
first, then
the tens.

Move on to place value counters

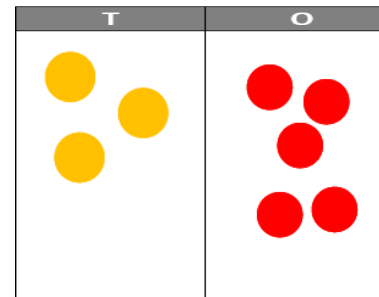


Pictorial

Children move on to drawing the counters using a tens frame.



Or a place value chart



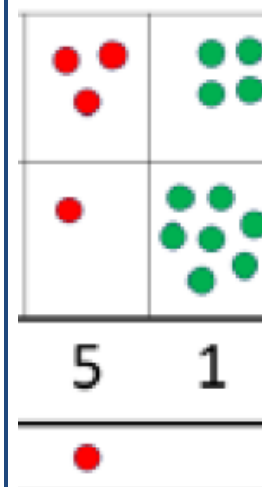
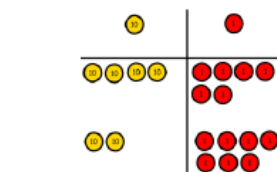
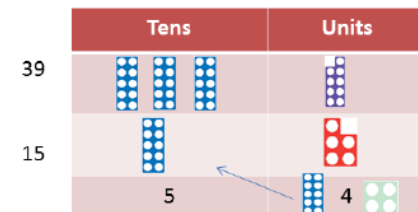
Abstract

Add the ones first, then the tens and then the hundreds.

$$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$$

Column addition
with regrouping or
exchanges

Exchange ten ones for a ten. Model using numicon and place value counters.



Children draw a representation of the grid to further support their understanding, carrying the ten underneath the line

Start by partitioning the numbers.

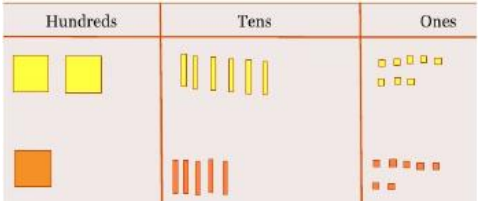
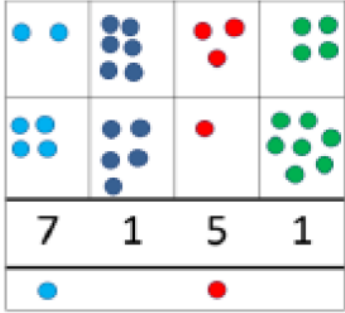
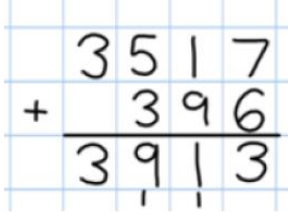
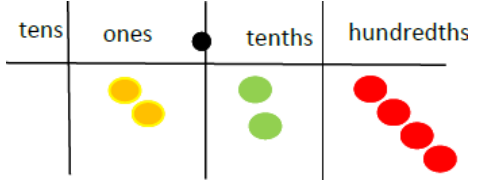
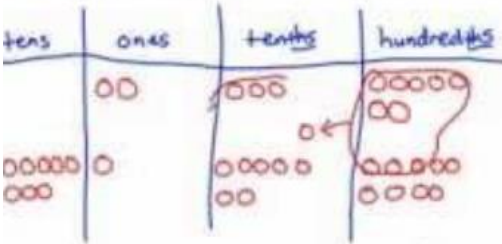
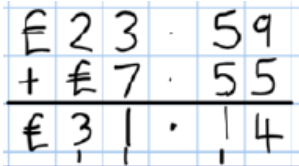
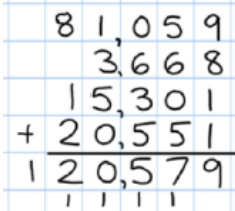
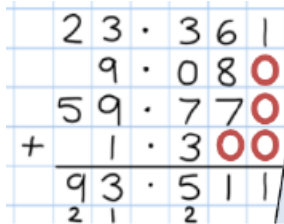
$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

Then
move
on to

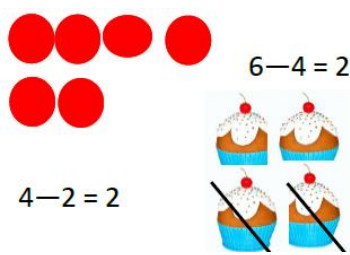
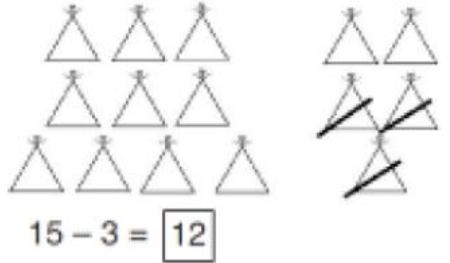
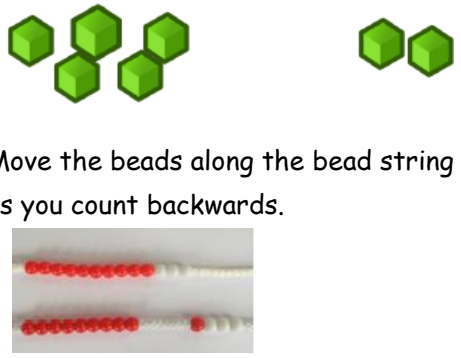
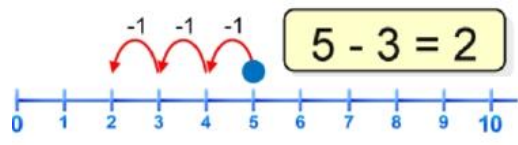
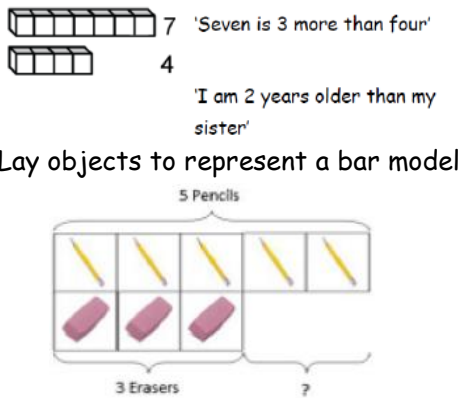
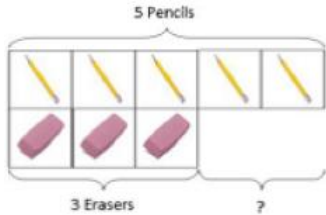
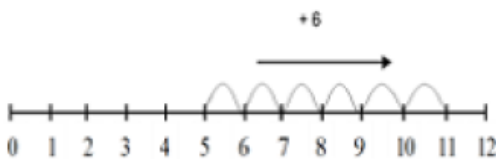
$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

formal column to show the exchange.

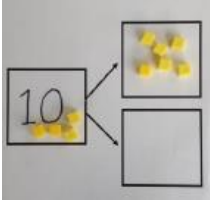
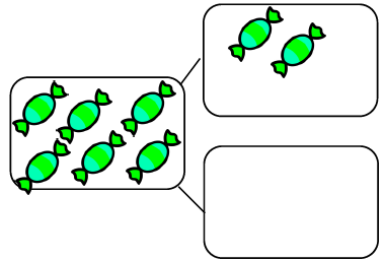
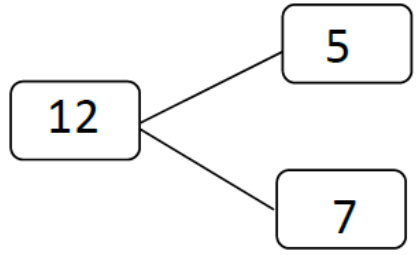

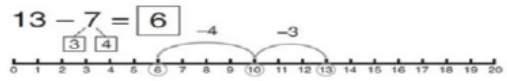
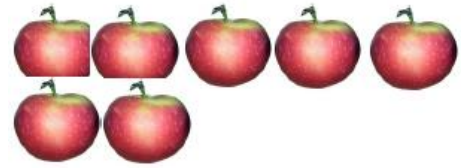

Y4-6 Addition +

| Objective & Strategy | Concrete | Pictorial | Abstract |
|---------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Y4 - add numbers with up to 4 digits</p> | <p>Children continue to use base 10 or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p>  | <p>Draw representations using pv grid.</p>  | <p>Continue from previous work to carry hundreds as well as tens.</p>  <p>Relate to money.</p> |
| <p>Y5 - add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p> | <p>As Year 4.</p>  <p>Introduce decimal place value counters and model exchange for addition.</p> | <p>2.37 + 81.79</p>  | <p>72.8</p> <p>+ 54.6</p> <p><u>127.4</u></p> <p>11</p>  |
| <p>Y6 - add several numbers of increasing complexity.</p> <p>Including adding money, measure and decimals with different numbers of decimal points.</p> | <p>As Year 5</p> | <p>As Year 5</p> |  <p>Insert zeros for place holders.</p>  |

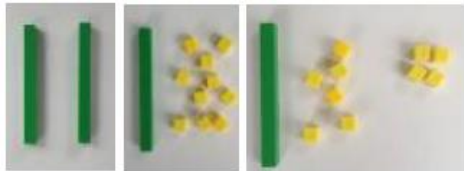
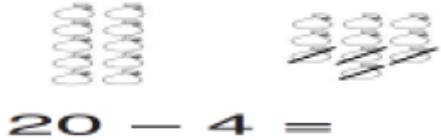


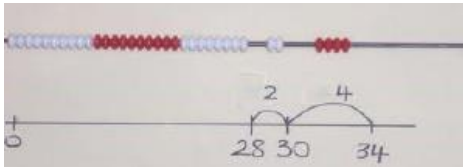
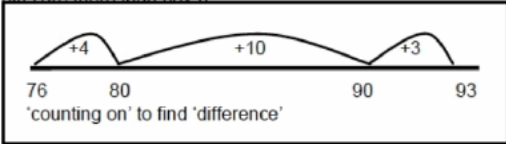
Y1 Subtraction -

| Objective & Strategy | Concrete | Pictorial | Abstract |
|----------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| Taking away ones. | <p>Use physical objects, counters, cubes, etc to show how objects can be taken away.</p>  | <p>Cross out drawn objects to show what has been taken away.</p>  | $7 - 4 = 3$ $16 - 9 = 7$ |
| Counting back. | <p>Move objects away from a group.</p>  <p>Move the beads along the bead string as you count backwards.</p> | <p>Count back in ones using a number line.</p>  | <p>Put 13 in your head, count back 4. What number are you at?</p> |
| Find the difference. | <p>Compare objects and amounts.</p>  <p>Lay objects to represent a bar model.</p>  | <p>Count on using a number line to find the difference.</p>  | <p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?</p> |

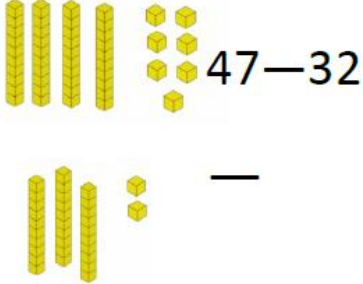
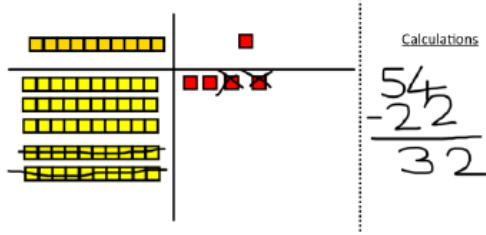
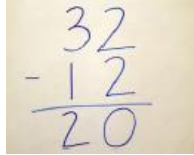
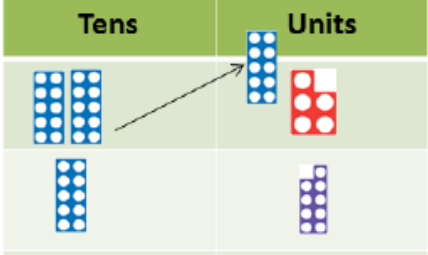
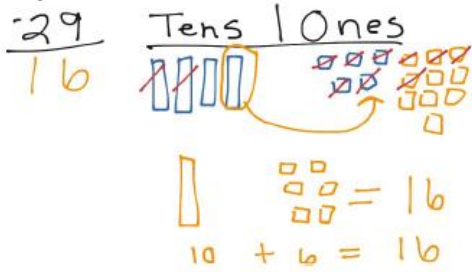
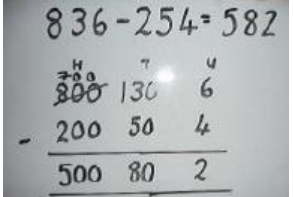
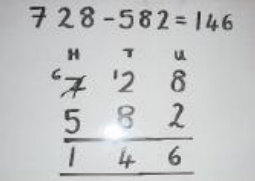
Y1 Subtraction -

| Objective & Strategy | Concrete | Pictorial | Abstract | | |
|--------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|---|
| <p>Represent and use number bonds and related subtraction facts within 20.</p> <p>Part Part Whole models</p> | <p>Link to addition. Use PPW model to model the inverse.</p>  <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> $10 - 6 = 4$ | <p>Use pictorial representations to show the part.</p>  | <p>Move to using numbers within the Part Part Whole model.</p>  | | |
| <p>Make 10</p> | <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken away 5.</p> $14 - 9$  | <p>Use a number line. Jump back 3 first, then another 4. Use ten as the stopping point.</p> $13 - 7$  | <p>How many do we take off first to get to 10? How many left to take off?</p> $16 - 8 =$ | | |
| <p>Bar model</p> |  $5 - 2 = 3$ |  | <table border="1" data-bbox="1680 965 2150 1053"> <tr> <td style="text-align: center;">8</td> <td style="text-align: center;">2</td> </tr> </table> $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$ | 8 | 2 |
| 8 | 2 | | | | |

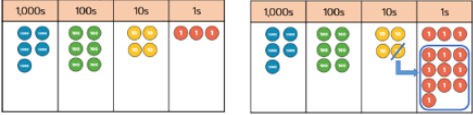
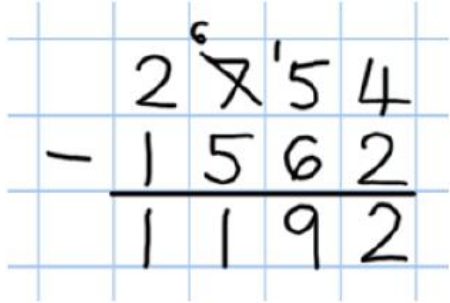
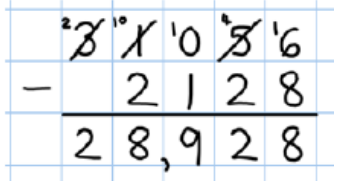
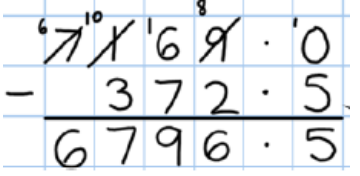
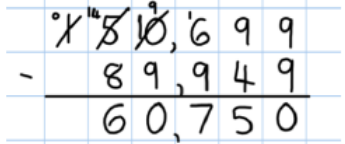
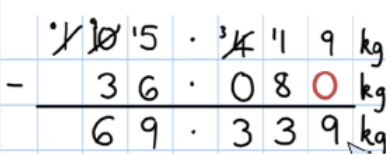
Y2 Subtraction -

| Objective & Strategy | Concrete | Pictorial | Abstract |
|------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| <p>Regroup a ten into ten ones</p> | <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>  |  | $20 - 4 = 16$ |
| <p>Partitioning to subtract without regrouping</p> <p>'friendly numbers'</p> | <p>$34 - 13 = 21$</p> <p>Use base 10 to show how to partition the number when subtracting without regrouping.</p>  | <p>Children draw representations of base 10 and cross off.</p>  $43 - 21 = 22$ | $43 - 21 = 22$ |
| <p>Make ten strategy</p> <p>Progression should be crossing one ten, crossing more than one ten, crossing hundreds.</p> | <p>Use a bead string to model counting to next ten and the rest.</p>  | <p>Use a number line to count on or back to the next ten and then the rest.</p>  <p>Some children can be confused with 'counting on' when subtracting therefore it is vital that children are taught to recognize that subtracting is 'finding the difference' so both counting on or back are valid methods.</p> | $93 - 76 = 17$ |

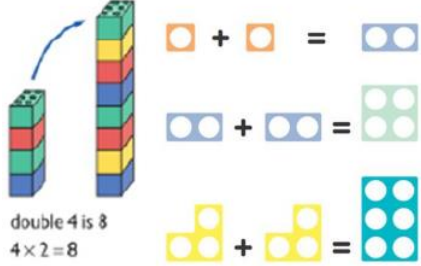

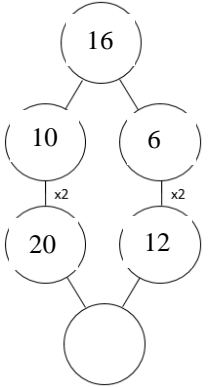
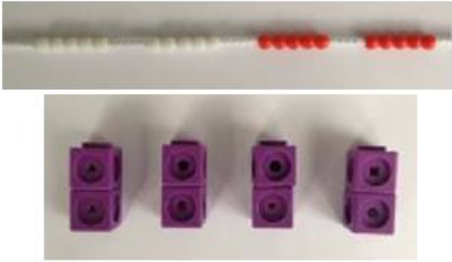
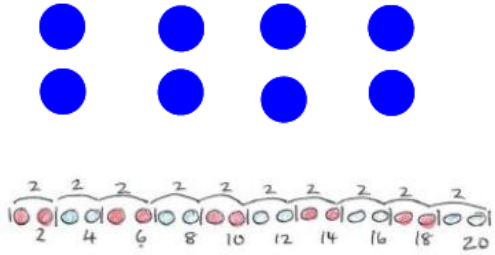
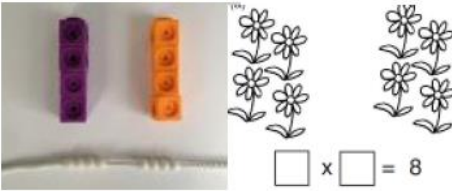

Y3 Subtraction -

| Objective & Strategy | Concrete | Pictorial | Abstract |
|-----------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Column subtraction without regrouping (friendly numbers)</p> | <p>Use base 10 or Numicom to model</p>  | <p>Draw representations to show understanding</p>  | <p>Intermediate step may be needed to lead to clear subtraction understanding</p> $47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$  |
| <p>Column subtraction with regrouping</p> | <p>Begin with base 10 or Numicom. Move to PV counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make...' for exchange.</p>  | <p>Children may draw base 10 or PV counters and cross then off.</p> $\begin{array}{r} 45 \\ - 29 \\ \hline 16 \end{array}$  | <p>Begin by partitioning into PV columns.</p> $836 - 254 = 582$  <p>Then move to formal method.</p> $728 - 582 = 146$  |
| | | | |

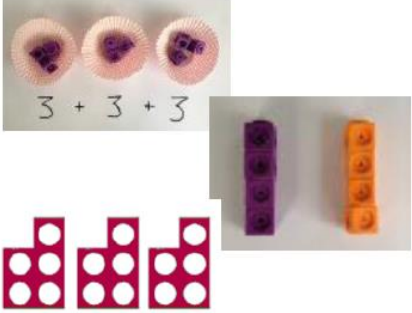
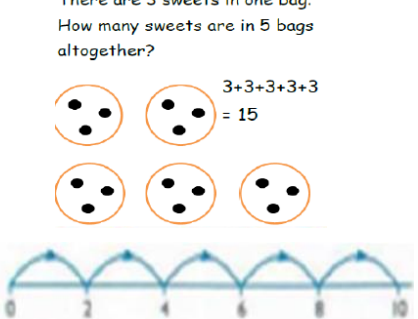

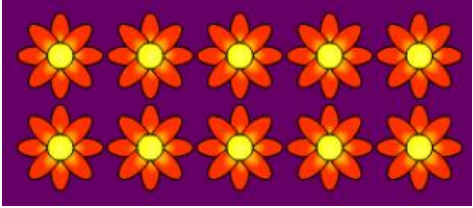
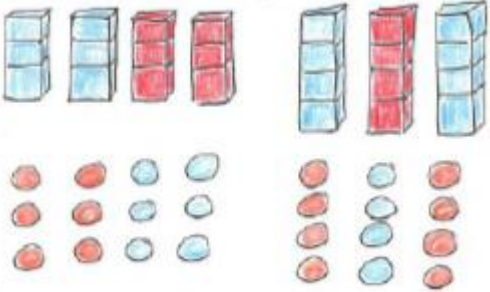
Y4-6 Subtraction

| Objective & Strategy | Concrete | Pictorial | Abstract |
|---------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Subtracting tens and ones.</p> <p>Year 4: Subtract with up to 4 digits</p> <p>Introduce decimal subtraction through context of money</p> | <p>Model process of exchange using Numicon, base 10 and then move into PV counters.</p> <p style="text-align: center;">$5,643 - 4,316 =$</p>  | <p>Children to draw PV counters and show their exchange (see Year 3).</p> | <p>Use the phrase 'take and make...' for exchange.</p>  |
| <p>Year 5: Subtract with at least 4 digits, including money and measures.</p> | <p>As Year 4.</p> | <p>Children to draw PV counters and show their exchange (see Year 3).</p> |  <p>Use zeros for placeholders.</p>  |
| <p>Year 6: Subtract with increasingly large and more complex numbers and decimal values.</p> | | |   |

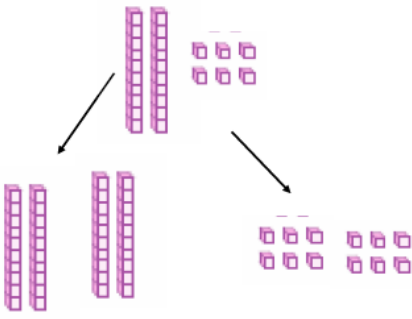
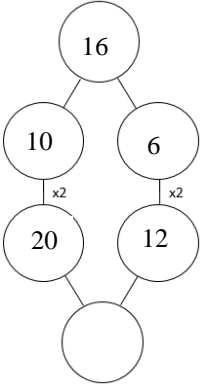
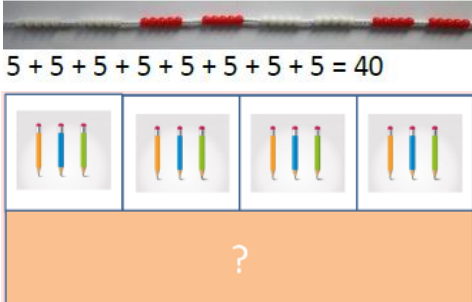
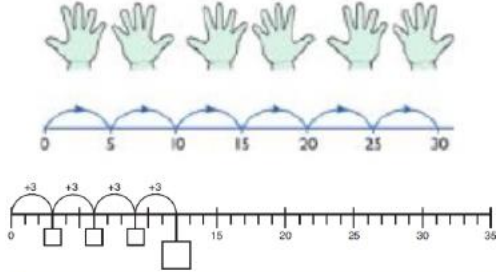
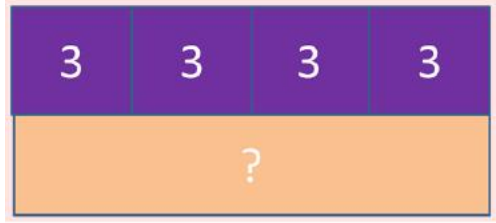
Y1 Multiplication

| Objective & Strategy | Concrete | Pictorial | Abstract |
|---------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Doubling</p> | <p>Use practical activities using manipulatives including cubes and Numicom to demonstrate doubling.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p> | <p>Draw pictures to show how to double numbers.</p> <p>Double 4 is 8</p>  | <p>Partition a number and then double each part before recombining it back together.</p>  |
| <p>Counting in multiples</p> | <p>Count the groups as children are skip-counting, children may use their fingers as they are skip-counting.</p>  | <p>Children make representations to show counting in multiples.</p>  | <p>Count in multiples of a number aloud. Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p> |
| <p>Making equal groups and counting the total</p> | <p>Use manipulatives to create equal groups.</p>  <p>$\square \times \square = 8$</p> | <p>Draw and make representations.</p> <p>Draw  to show $2 \times 3 = 6$</p> | <p>$2 \times 4 = 8$</p> |

Y1 Multiplication

| Objective & Strategy | Concrete | Pictorial | Abstract |
|----------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Repeated addition | Use different objects to add equal groups.  $3 + 3 + 3$ | Use pictorial including number lines to solve problems. There are 3 sweets in one bag. How many sweets are in 5 bags altogether? $3+3+3+3+3 = 15$  | Write addition sentences to describe objects and pictures.  $2 + 2 + 2 + 2 + 2 = 10$ |
| Understanding arrays | Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 5, etc.  | Draw representations of arrays to show understanding.  | $3 \times 2 = 6$ $2 \times 5 = 10$ |
| | | | |

Y2 Multiplication

| Objective & Strategy | Concrete | Pictorial | Abstract |
|------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Doubling</p> | <p>Model doubling using base 10 and PV counters.</p>  <p>$40 + 12 = 52$</p> | <p>Draw pictures and representations to show how to double numbers</p> | <p>Partition a number and then double each part before recombining it back together.</p>  |
| <p>Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)</p> | <p>Count the groups as children are skip-counting, children may use their fingers as they are skip-counting. Use bar models.</p>  <p>$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$</p> | <p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p>   | <p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15, 0, 5, 10, 15, 20, 25, 30</p> <p>$4 \times 3 = \square$</p> |

Y2 Multiplication

Objective & Strategy

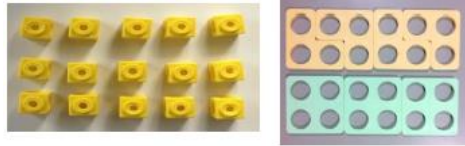
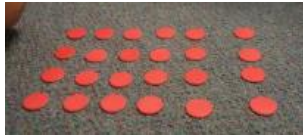
Concrete

Pictorial

Abstract

Multiplication is commutative.

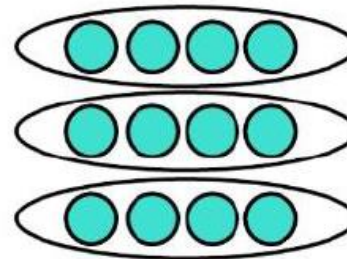
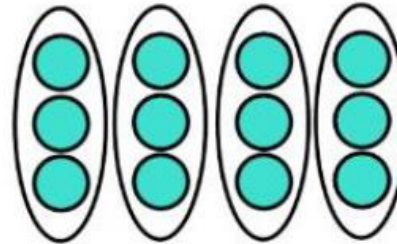
create arrays using counters and cubes and Numicom.



Pupils should understand that an array can represent different equations and that as multiplication is commutative the order of the multiplication does not affect the answer.



Use representations of arrays to show different calculations and explore commutativity.



$$12 = 3 \times 4$$

$$12 = 4 \times 3$$

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Using the inverse.

This should be taught alongside division so pupils learn how they work alongside each other.



\times =
 \times =
 \div =
 \div =

$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$

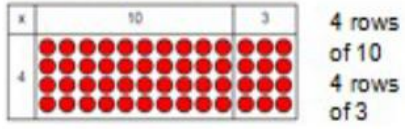
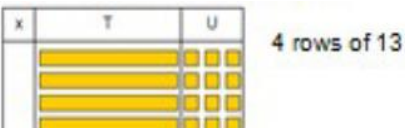

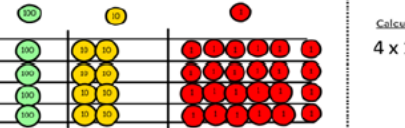
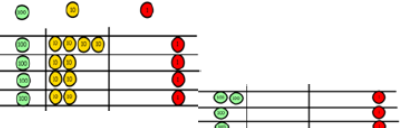
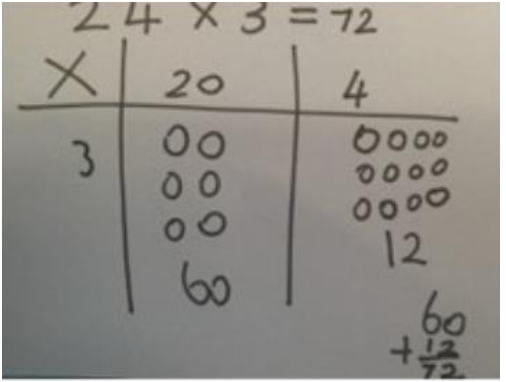
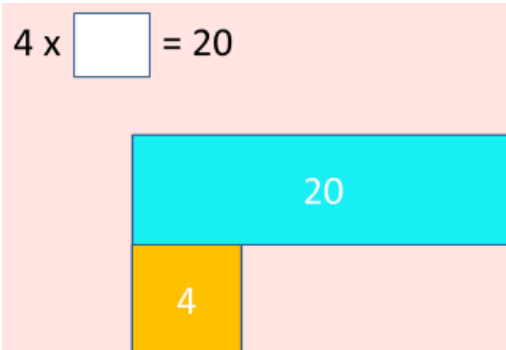
$$8 = 4 \times 2$$

$$2 = 8 \div 4$$

$$4 = 8 \div 2$$

Show all 8 related fact family sentences.

Y3 Multiplication

| Objective & Strategy | Concrete | Pictorial | Abstract | | | | | | | | | | | | | | | |
|----------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----|---|---|-----|----|--|----|---|----|-----|----|---|----|----|
| <p>Grid method</p> | <p>Show the links with arrays to first introduce the grid method.</p>  <p>Move on to base 10 to move towards a more compact method. Move on to PV counters to show how</p>  <p>we are finding groups of a number. We are multiplying by 4 so we need 4 rows. Fill each row with 126</p>  <p>Fill each row with 126</p>  <p>Add up each column, starting with the ones making any exchanges needed</p>  <p>Then you have your answer</p> | <p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p>  | <p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1713 287 2172 422"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2-digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="1713 798 2172 1101"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table> | x | 30 | 5 | 7 | 210 | 35 | | 10 | 8 | 10 | 100 | 80 | 3 | 30 | 24 |
| x | 30 | 5 | | | | | | | | | | | | | | | | |
| 7 | 210 | 35 | | | | | | | | | | | | | | | | |
| | 10 | 8 | | | | | | | | | | | | | | | | |
| 10 | 100 | 80 | | | | | | | | | | | | | | | | |
| 3 | 30 | 24 | | | | | | | | | | | | | | | | |

Y4 Multiplication

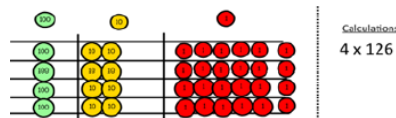
Objective & Strategy

Grid method recap from year 3 for 2-digits x 1-digit

Move to multiplying 3-digit numbers by 1-digit. (year 4 expectation)

Concrete

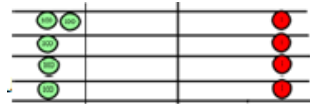
Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Fill each row with 126



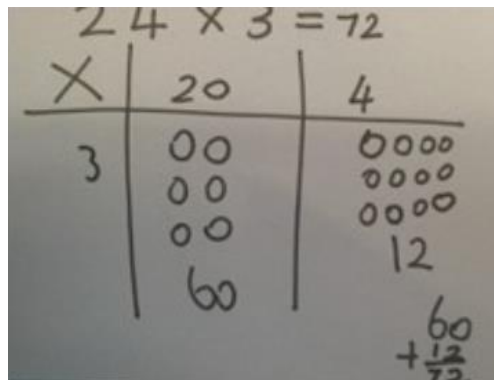
Add up each column, starting with the ones making any exchanges needed.



Pictorial

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their



Abstract

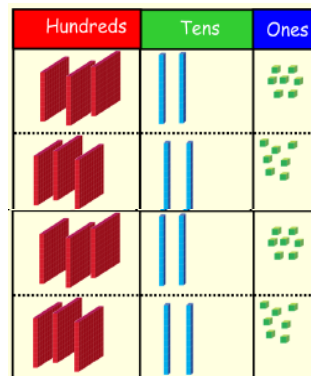
Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

| | | |
|---|-----|----|
| x | 30 | 5 |
| 7 | 210 | 35 |

$$210 + 35 = 245$$

Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$



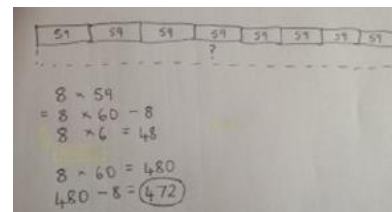
It is important at this stage that they always multiply the ones first.

The corresponding long multiplication is modelled alongside.

| | | | |
|---|------|----|----|
| x | 300 | 20 | 7 |
| 4 | 1200 | 80 | 28 |



The grid method may be used to show how this relates to a formal written method.



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

$$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$$



| | | | |
|---|---|---|---|
| | 3 | 2 | 7 |
| x | | | 4 |
| | 1 | 3 | 0 |
| | | 1 | 2 |

This may lead to a compact method.

Y5-6

Objective & Strategy

Column multiplication for 3 and 4-digit \times 1-digit.

Concrete

| Hundreds | Tens | Ones |
|----------|------|------|
| | | |
| | | |
| | | |
| | | |

It is important at this stage that they always multiply the ones first.

Children can continue to be supported by PV counters at the stage of multiplication. This initially is done where there is no regrouping.

$$321 \times 2 = 642$$

Pictorial

| | | | |
|----------|------|----|----|
| \times | 300 | 20 | 7 |
| 4 | 1200 | 80 | 28 |



$$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$$



| | | | |
|----------|---|---|---|
| | 3 | 2 | 7 |
| \times | | | 4 |
| | 1 | 3 | 0 |
| | | 2 | 8 |

This may lead to a compact method.

Column multiplication

Manipulatives may still be used with the corresponding long multiplication modelled alongside.

| | 10 | 8 |
|----|-----|----|
| 10 | 100 | 80 |
| 3 | 30 | 24 |



| | | | |
|----------|---|---|---|
| | | 1 | 8 |
| \times | | 1 | 3 |
| | | 5 | 4 |
| | | 2 | |
| | 1 | 8 | 0 |
| | 2 | 3 | 4 |

18×3 on the first row

($8 \times 3 = 24$, carrying the 2 for 20, then 1×3)

| | | | | |
|----------|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| \times | | | 1 | 6 |
| | 7 | 4 | 0 | 4 |
| | 1 | 2 | 3 | 4 |
| | 1 | 9 | 7 | 4 |

18×10 on the 2nd row. Show multiplying by 10 by putting zero in units first

Y6 Multiplication

Objective & Strategy

Multiplying decimals up to 2 decimal places by a single digit.

Concrete

Pictorial

Abstract

Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.

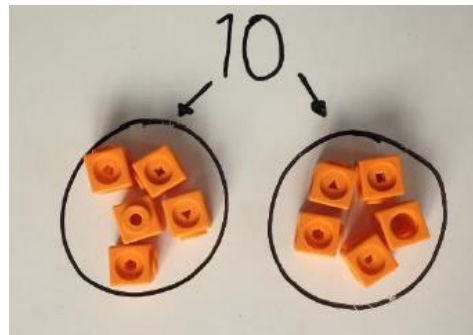
$$\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \\ \hline \end{array}$$

Y1 Division

Objective & Strategy

Division is sharing

Concrete



I have 10 cubes, can you share them equally in 2 groups?

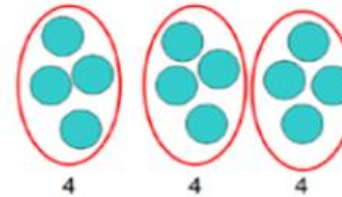
Pictorial

Children use pictures or shapes to share quantities.



8 shared between 2 is 4

Sharing:

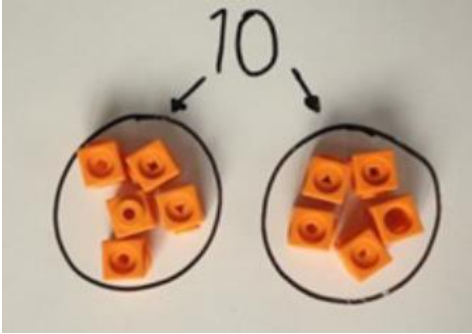

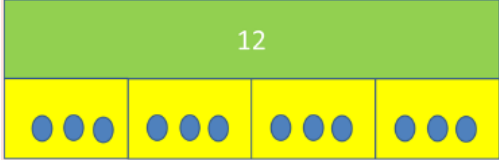
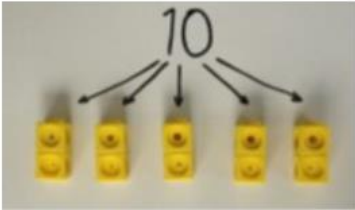
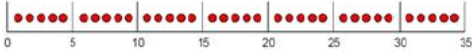
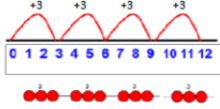
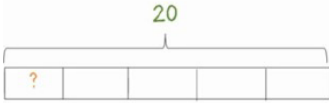


12 shared between 3 is 4

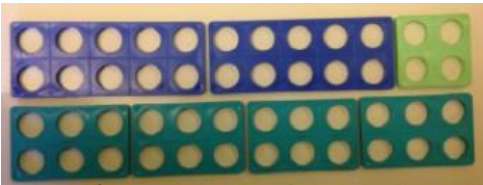
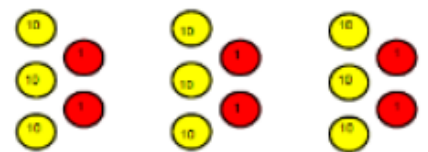
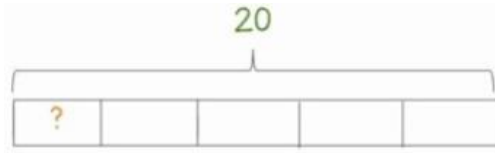

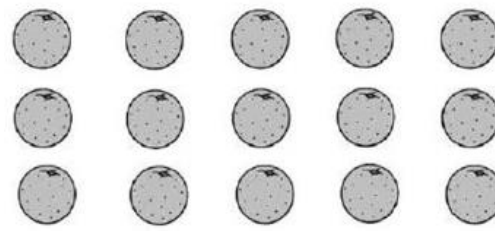
Abstract

12 shared between
3 is 4

Y2 Division

| Objective & Strategy | Concrete | Pictorial | Abstract |
|----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|
| Division as sharing |  <p>I have 10 cubes, can you share them equally in 2 groups?</p> | <p>Children use pictures or shapes to share quantities.</p>  <p>$8 \div 2 = 4$</p> <p>Children use bar modelling to show and support understanding.</p>  <p>$12 \div 4 = 3$</p> | <p>$12 \div 4 = 3$</p> |
| Division as grouping | <p>Divide quantities into equal groups. use cubes, counters, objects or PV counters to aid understanding.</p>   | <p>Use number lines for grouping.</p>  <p>$12 \div 3 = 4$</p> <p>Think of the bar as a whole split into the number of groups you are dividing by and work out how many would be within each group.</p>  <p>$20 \div 5 = ?$ $5 \times ? = 20$</p> | <p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p> |

Y3 Division

| Objective & Strategy | Concrete | Pictorial | Abstract |
|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Division as grouping</p> | <p>Use cubes, counters, objects or PV counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$  | <p>Continue to use bar modelling to aid solving division problems.</p>  <p>20</p> $20 \div 5 = ?$ $5 \times ? = 20$ | <p>How many groups of 6 in 24?</p> $24 \div 6 = 4$ |
| <p>Division with arrays</p> |  <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>E.g. $15 \div 3 = 5$ $5 \times 3 = 15$</p> $15 \div 5 = 3$ $3 \times 5 = 15$ | <p>Draw an array and use lines to split the array into groups to make multiplication and division sentences.</p>  | <p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$ |

Y3 Division

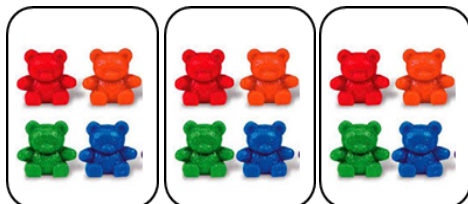
Objective & Strategy

Division with remainders

Concrete

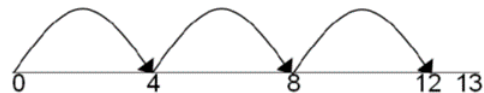
$$14 \div 3 =$$

Divide objects between groups and see how much is left over.



Pictorial

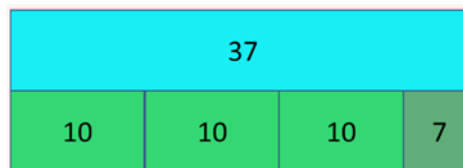
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



Use bar model to show division with remainders.



Abstract

Complete written divisions and show the remainder using r

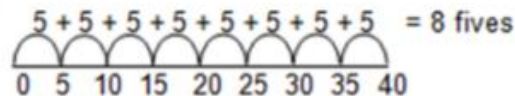
$$29 \div 8 = 3 \text{ REMAINDER } 5$$

↑ ↑ ↑ ↑
 dividend divisor quotient remainder

Example without remainder:

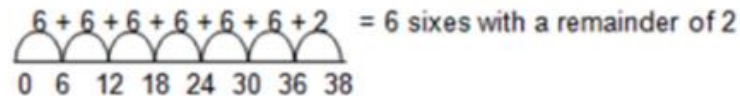
$$40 \div 5$$

Ask "How many 5s in 40?"



Example with remainder:

$$38 \div 6$$



For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Y4-6 Division

Objective & Strategy

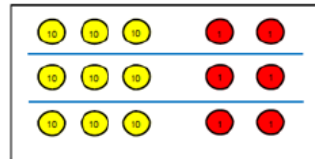
Divide at least 3-digit numbers by 1-digit

Short division

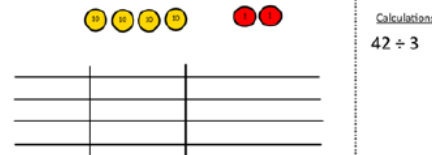
Concrete

$$96 \div 3$$

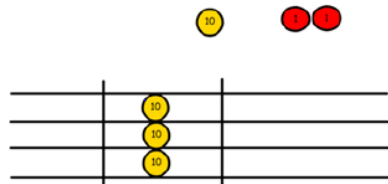
| | Tens | Units |
|--|------|-------|
| | 3 | 2 |



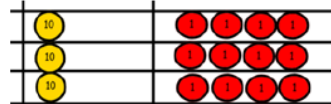
Use place value counters to divide using the bus stop method alongside
 $42 \div 3 =$



Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



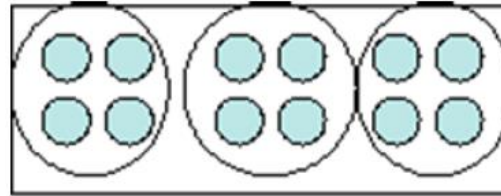
We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Pictorial

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Abstract

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$$

$$\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 53509} \end{array}$$

Long Division

Step 1 - a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Y6 Division

Long Division

Step 1 continued...

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Y6 Division

Y6 Division

Long Division

Step 2 - a remainder in the tens

| 1. Divide. | 2. Multiply & subtract. | 3. Drop down the next digit. |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{4} \\ 18 \end{array}$ <p>Two goes into 5 two times, or 5 tens $\div 2 = 2$ whole tens -- but there is a remainder!</p> | $\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{-4} \\ 1 \end{array}$ <p>To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.</p> | $\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p> |

| 1. Divide. | 2. Multiply & subtract. | 3. Drop down the next digit. |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p> | $\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.</p> | $\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p> |

Long Division

Step 2 - a remainder in any of the place values

Y6 Division

| | | |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Divide. $\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.</p> | 2. Multiply & subtract. $\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ \underline{-2} \\ 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p> | 3. Drop down the next digit. $\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ \underline{-2} \downarrow \\ 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p> |
| Divide. $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p> | Multiply & subtract. $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p> | Drop down the next digit. $\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p> |
| 1. Divide. $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p> | 2. Multiply & subtract. $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p> | 3. Drop down the next digit. $\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p> |

Long Division

Chunking method

Although the chunking method is a stronger algorithm to mathematically model or explain division, historically pupils made more errors using this method than the old 'bus-stop' method long and short division.

However, this method can be useful to those children who are confident using short division when dividing by 2-digit numbers.

division by chunking

$$216 \div 12 = 18$$

$$\begin{array}{r} 12 \overline{) 216} \\ \underline{(10 \times 12) - 120} \\ 096 \\ \underline{(5 \times 12) - 60} \\ 36 \\ \underline{(3 \times 12) - 36} \\ 00 \end{array}$$

← children must be confident in subtraction for this method.

How many $\times 12$ altogether?

$$10 + 5 + 3 = 18$$

of

$$24 \overline{) 556} \quad 23 \text{ r } 4$$

for

$$\underline{-480} \quad 24 \times 20$$

$$76$$

not

$$\underline{-72} \quad 24 \times 3$$

$$4$$

Y6 Division